

Corollary of Baye's Theorem

Research Article

S.Vijayakumar^{1*}, V.Archana¹ and M.Kalaivanan¹

¹ Department of Mathematics, P.G. Extension Centre, Bharathidasan University, Perambalur, Tamilnadu, India.

Abstract: We have worked out a corollary based on “Baye’s theorem”. From this I have extracted mutually disjoint events, and A and B are two arbitrary events. These events are subsets. Through this $P[(E_i)/(AB)]$ also satisfies, Baye’s Theorem.

Keywords: Probability, mutually disjoint events, future events.

© JS Publication.

1. Introduction and Preliminary

Baye’s Theorem introduced by Thomas Bayes, a British Mathematician, provides a means for making these probability calculations. In the discussion of conditional probability we indicated that revising probability when new information is obtained is an important phase of probability analysis. We begin our analysis with initial or prior probability estimates for specific events of interest. Then, from sources such as a sample, a special report, a product test, and so on we obtain some additional information about the events.

In this paper, results are presented that hold for two arbitrary events, and are useful in the graphs. The first is obvious and appears in baye’s Theorem is extensively used in statistical inference, and by business and management executives in arriving at valid decisions in the face of uncertainty.

2. Main result

Theorem 2.1. If E_1, E_2, \dots, E_n are mutually disjoint events, with $P(E_i) \neq 0$, ($i = 1, 2, \dots, n$) then, for any arbitrary event A and B which is a subset of $\bigcup_{i=1}^n E_i$ such that $P(A) > 0$ and $P(B) > 0$, we have

$$P(E_i/(A \cup B)) = \frac{P(E_i)[P(A/E_i) + P(B/E_i)]}{\sum_{i=1}^n P(E_i)[P(A/E_i) + P(B/E_i)]} \quad (or)$$

$$P(E_i/A \cup B) = \frac{P(E_i)[P(A/E_i) + P(B/E_i)]}{P(A \cup B)}$$

Proof. Since $A \subset \bigcup_{i=1}^n E_i$ and $B \subset \bigcup_{i=1}^n E_i$. Therefore,

$$(A \cup B) = (A \cup B) \cap \bigcup_{i=1}^n E_i$$

$$(A \cup B) = \bigcup_{i=1}^n ((A \cup B) \cap E_i)$$

* E-mail: mathematicianvijayakumar@gmail.com

Taking probability on both side,

$$\begin{aligned}
 P(A \cup B) &= P\left\{\bigcup_{i=1}^n [(A \cup B) \cap E_i]\right\} \\
 &= \sum_{i=1}^n P[(A \cup B) \cap E_i] \\
 &= \sum_{i=1}^n P[(A \cap E_i) \cup (B \cap E_i)] && \text{(by distributive law)} \\
 &= \sum_{i=1}^n \{P(A \cap E_i) + P(B \cap E_i)\} && \text{(by addition theorem of probability)} \quad (1)
 \end{aligned}$$

Also we have

$$P(A \cap E_i) = P(E_i) \cdot P(A/E_i) \quad (a)$$

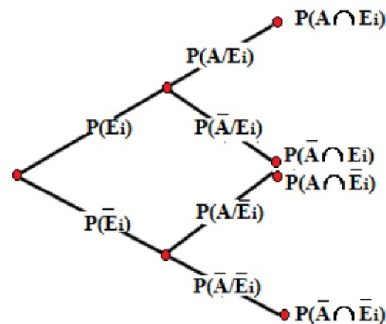
$$P(B \cap E_i) = P(E_i) \cdot P(B/E_i) \quad (b)$$

Applying (a) and (b) in (1)

$$\begin{aligned}
 P(A \cup B) &= \sum_{i=1}^n [P(E_i)P(A/E_i) + P(E_i)P(B/E_i)] \\
 P(A \cup B) &= \sum_{i=1}^n P(E_i)[P(A/E_i) + P(B/E_i)] \quad (2)
 \end{aligned}$$

Also we have

$$\begin{aligned}
 P((A \cup B) \cap E_i) &= P(A \cup B)P(E_i/A \cup B) \\
 \Rightarrow P(E_i/A \cup B) &= \frac{P((A \cup B) \cap E_i)}{P(A \cup B)} \\
 &= \frac{P[(A \cap E_i) \cup (B \cap E_i)]}{P(A \cup B)} && \text{(By distributive law of probability)} \\
 &= \frac{P[(A \cap E_i) + (B \cap E_i)]}{P(A \cup B)} && \text{(By addition theorem of Probability)} \\
 &= \frac{P(E_i)P(A/E_i) + P(E_i)P(B/E_i)}{P(A \cup B)} && \text{(Multiplication theorem of probability)} \\
 &= \frac{P(E_i)[P(A/E_i) + P(B/E_i)]}{\sum_{i=1}^n P(E_i)[P(A/E_i) + P(B/E_i)]} && \text{(by Equation (2))}
 \end{aligned}$$

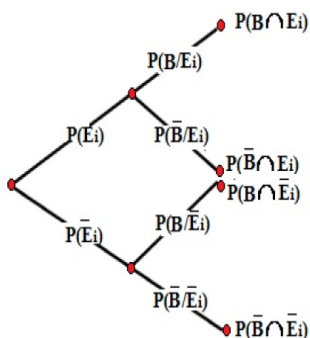


$$P(A \cap E_i) = P(E_i)P(A/E_i)$$

$$P(\bar{A} \cap E_i) = P(E_i)P(\bar{A}/E_i)$$

$$P(A \cap \bar{E}_i) = P(\bar{E}_i)P(A/\bar{E}_i)$$

$$P(\bar{A} \cap \bar{E}_i) = P(\bar{E}_i)P(\bar{A}/\bar{E}_i)$$

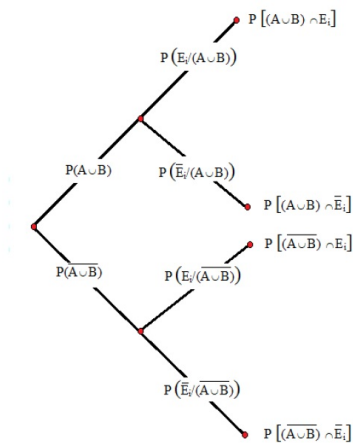


$$P(B \cap E_i) = P(E_i)P(B/E_i)$$

$$P(\bar{B} \cap E_i) = P(E_i)P(\bar{B}/E_i)$$

$$P(B \cap \bar{E}_i) = P(\bar{E}_i)P(B/\bar{E}_i)$$

$$P(\bar{B} \cap \bar{E}_i) = P(\bar{E}_i)P(\bar{B}/\bar{E}_i)$$

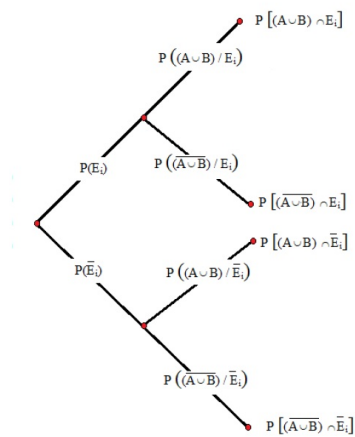


$$P[(A \cup B) \cap E_i] = P(A \cup B)P(E_i/(A \cup B))$$

$$P[(A \cup B) \cap \bar{E}_i] = P(A \cup B)P(\bar{E}_i/(A \cup B))$$

$$P[(\overline{A \cup B}) \cap E_i] = P(\overline{A \cup B})P(E_i/(\overline{A \cup B}))$$

$$P[(\overline{A \cup B}) \cap \bar{E}_i] = P(\overline{A \cup B})P(\bar{E}_i/(\overline{A \cup B}))$$

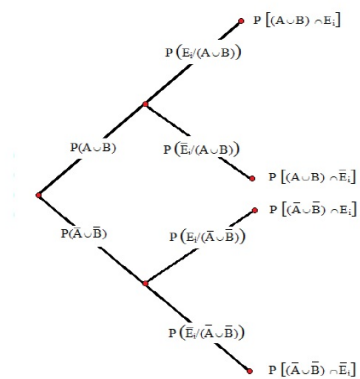


$$P[(A \cup B) \cap E_i] = P(E_i)P((A \cup B)/E_i)$$

$$P[(\overline{A \cup B}) \cap E_i] = P(E_i)P(\overline{A \cup B}/E_i)$$

$$P[(A \cup B) \cap \bar{E}_i] = P(\bar{E}_i)P((A \cup B)/\bar{E}_i)$$

$$P[(\overline{A \cup B}) \cap \bar{E}_i] = P(\bar{E}_i)P(\overline{A \cup B}/\bar{E}_i)$$

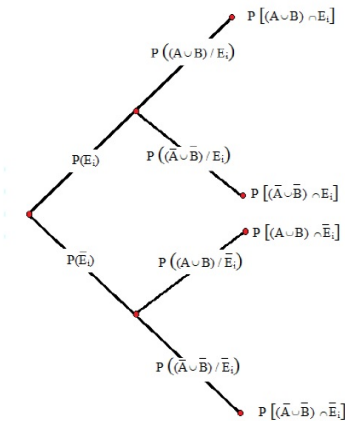


$$P[(A \cup B) \cap E_i] = P(A \cup B)P(E_i/(A \cup B))$$

$$P[(A \cup B) \cap \bar{E}_i] = P(A \cup B)P(\bar{E}_i/(A \cup B))$$

$$P[(\bar{A} \cup \bar{B}) \cap E_i] = P(\bar{A} \cup \bar{B})P(E_i/(\bar{A} \cup \bar{B}))$$

$$P[(\bar{A} \cup \bar{B}) \cap \bar{E}_i] = P(\bar{A} \cup \bar{B})P(\bar{E}_i/(\bar{A} \cup \bar{B}))$$



$$P[(A \cup B) \cap E_i] = P(E_i)P((A \cup B)/E_i)$$

$$P[(\bar{A} \cup \bar{B}) \cap E_i] = P(E_i)P((\bar{A} \cup \bar{B})/E_i)$$

$$P[(A \cup B) \cap (\bar{E}_i)] = P(\bar{E}_i)P((A \cup B)/\bar{E}_i)$$

$$P[(\bar{A} \cup \bar{B}) \cap (\bar{E}_i)] = P(\bar{E}_i)P((\bar{A} \cup \bar{B})/\bar{E}_i)$$

□

References

- [1] Gupta, S.C. and Kapoor, V.K., *Fundamentals of Mathematical Statistics*, Sultan chand and sons Publishers, New Delhi, India. (2006).
- [2] Vittal, P.R., *Mathematical Statistics*, Margham Publications, Chennai, (2004).