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Edge Degree Sequence of a Fuzzy Graph

Research Article

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Abstract: In this paper, edge degree sequence of a fuzzy graph is introduced and some of its properties are studied. Necessary and sufficient conditions for a sequence of two or three real numbers to be an edge degree sequence of a fuzzy graph are obtained. Also necessary and sufficient conditions for a finite sequence of real numbers to be an edge degree sequence of a fuzzy graph on a path and cycle are obtained.

MSC: 05C07, 05C38.

Keywords: Degree of an edge, edge degree sequence of a fuzzy graph.

1. Introduction

Graph theory serves as a mathematical model for any system involving binary relation. Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [6]. Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of neural networks, expert systems, information theory, cluster analysis, medical diagnosis, control theory etc. Rosenfeld [6] has obtained the fuzzy graph theoretic concepts like bridges paths, cycles, trees and connectedness and established some of their properties. Fundamental sequence of a fuzzy graph is defined in [7]. A. Nagoor gani and K. Radha introduced incidence sequence of a fuzzy graph in [8]. Degree of an edge in a fuzzy graph is defined by K. Radha and N. Kumaravel in [9]. In this paper we discussed about the edge degree sequence of fuzzy graph. We also determine the conditions for a sequence of real numbers to be an edge degree sequence of a fuzzy graph. First we go through some basic concepts which can be seen in [1–9]. For the basic concepts in crisp graph theory, we consider [3].

Definition 1.1 ([6]). *A fuzzy graph G is a pair of functions $G : (\sigma, \mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ (ie) $\mu(xy) \leq \sigma(x) \wedge \sigma(y) \quad \forall\, x, y \in V$. The underlying crisp graph of $G : (\sigma, \mu)$ is denote by $G^* : (V, E)$ where $E \subseteq V \times V$.*

Definition 1.2 ([4]). *The degree of a vertex u of a fuzzy graph G is the sum of membership values of edges of G which are incident at u $d(u) = \sum_{uv \in E} \mu(uv)$.*

Definition 1.3 ([8]). *In a fuzzy graph $G : (\sigma, \mu)$, the degree of an edge $e = uv \in V$ is $d(uv) = d(u) + d(v) - 2\mu(uv)$. G is an edge regular fuzzy graph if all the edges have the same edge degree.*

Definition 1.4 ([4]). *The size of a fuzzy graph G is $S(G) = \sum_{uv \in E} \mu(uv)$.*

Definition 1.5 ([5]). *A degree sequence in which no two of its elements are equal is called perfect degree sequence.*

Definition 1.6 ([5]). *A degree sequence in which exactly two of its elements are same is called quasi- perfect*

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2. Edge Degree Sequence of a Fuzzy Graph

Definition 2.1. *A sequence of real numbers $(d_1, d_2, d_3, \ldots, d_n)$ with $d_1 \geq d_2 \geq d_3 \geq \cdots \geq d_n$ where d_i's are the degrees of edges of a fuzzy graph G is the edge degree sequence of G, denoted by $eds(G)$.*

Example 2.2.

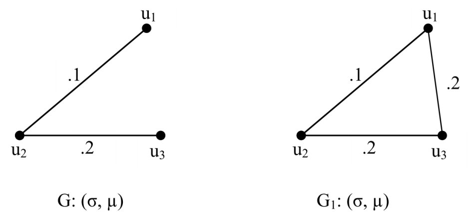

Figure 1:

Edge degree sequence of G is (.2, .1). Edge degree sequence of G_1 is (.4, .3, .3).

Definition 2.3. *A sequence $S = (d_1, d_2, d_3, \ldots, d_n)$ of real numbers is said to be fuzzy edge graphic sequence if there exists a graph G whose edges have degree d_i and G is called realization of S.*

Definition 2.4. *An edge degree sequence of real numbers in which no two of its elements are equal is called perfect edge degree sequence.*

The edge degree sequence of G in Example 2.2 is perfect.

Definition 2.5. *An edge degree sequence of real numbers in which exactly two of its elements are same is called quasi-perfect.*

The edge degree sequence of G_1 in Example 2.2 is quasi-perfect. We obtain some properties of edge degree sequence of a fuzzy graph in the following theorems.

Theorem 2.6. *An edge degree sequence of a fuzzy graph G is a constant sequence if and only if G is edge regular.*

Proof. An edge degree sequence of a fuzzy graph G is a constant sequence if and only if all the edges of G have same edge degree which happens if and only if G is edge regular. □

Theorem 2.7. *If $G : (\sigma, \mu)$ is a fuzzy graph, then the sum of all the terms of the edge degree sequence is $\sum_{uv \in E} d(uv) = \sum_{u \in V} d_{G^*}(u) d_G(u) - 2S(G)$.*

Proof. By the definition

$$\begin{aligned} \sum_{uv \in E} d(uv) &= \sum_{uv \in E} (d(u) + d(v) - 2\mu(uv)) \\ &= \sum_{uv \in E} (d(u) + d(v)) - 2 \sum_{uv \in E} \mu(uv) \end{aligned}$$

In $\sum_{uv \in E} (d(u) + d(v))$, each $d(u)$ appears $d_{G^*}(u)$ times as the summation runs over all edges $uv \in E$. Therefore $\sum_{uv \in E} d(uv) = \sum_{u \in V} (d_{G^*}(u) d_G(u)) - 2S(G)$. □

Theorem 2.8. *If $G : (\sigma, \mu)$ is a fuzzy graph such that $\sigma(u) = c$, $\forall\, u \in V$, then $d_G(e)$ is at most $c d_{G^*}(e)$, for every $e \in E$.*

Proof. Since $(u) = c \ \forall \ u \in V$, $\mu(uv) \leq c \ \forall \ uv \in E$. By the definition

$$\begin{aligned}\sum_{uv \in E} d_G(uv) &= \sum_{w \neq v} \mu(uw) + \sum_{w \neq u} \mu(vw) \\ &\leq c(d_{G^*}(u) - 1) + c(d_{G^*}(v) - 1) \\ &= c(d_{G^*}(u) + d_{G^*}(v)) - 2) \\ &= cd_{G^*}(e)\end{aligned}$$

□

Corollary 2.9. *If $G : (\sigma, \mu)$ is a fuzzy graph such that $\sigma(u) = c \ \forall \ u \in V$, then the sum of all the terms of the edge degree sequence of G is at most $c \sum_{e \in E} d_{G^*}(e)$.*

Corollary 2.10. *If $G : (\sigma, \mu)$ is a fuzzy graph on a cycle with n vertices such that $\sigma(u) = c \ \forall \ u \in V$ then edge degree $d_G(e) \leq 2c \ \forall \ e \in E$ and the sum of all the terms of the edge degree sequence of G is at most 2cn.*

Theorem 2.11. *If $G : (\sigma, \mu)$ is a fuzzy graph such that $\mu(uv) = c, \ \forall \ u \in V$, then $d_G(e)$ is $cd_{G^*}(e)$, for every $e \in E$.*

Proof. By the definition

$$\begin{aligned}\sum_{uv \in E} d_G(uv) &= \sum_{w \neq v} \mu(uw) + \sum_{w \neq u} \mu(vw) \\ &= c(d_{G^*}(u) - 1) + c(d_{G^*}(v) - 1) \\ &= c(d_{G^*}(u) + d_{G^*}(v)) - 2) \\ &= cd_{G^*}(e)\end{aligned}$$

□

Corollary 2.12. *If $G : (\sigma, \mu)$ is a fuzzy graph such that $\mu(uv) = c \ \forall \ u \in V$, then the sum of all the terms of the edge degree sequence of G is $c \sum_{e \in E} d_{G^*}(e)$.*

Corollary 2.13. *If $G : (\sigma, \mu)$ be a fuzzy graph on a cycle with n vertices such that $\mu(uv) = c \ \forall \ uv \in E$ then $d(e) = 2c \ \forall \ e \in E$ and the sum of all the terms of the edge degree sequence of G is 2cn.*

3. Characterizations

In this section, we obtain characterization for a given sequence of positive real numbers to be an edge degree sequence of a fuzzy graph. Then we illustrate the procedure with examples.

Theorem 3.1. *(d_1, d_2) is the edge degree sequence of a fuzzy graph if and only if $0 < d_1, d_2 \leq 1$.*

Proof. If (d_1, d_2) is the edge degree sequence of a fuzzy graph G, then G must be a path, say, $v_1v_2v_3$, on three vertices. Now

$$\begin{aligned}d(v_1v_2) &= d(v_1) + d(v_2) - 2\mu(v_1v_2) \\ &= \mu(v_1v_2) + \mu(v_1v_2) + \mu(v_2v_3) - 2\mu(v_1v_2) \\ &= \mu(v_2v_3)\end{aligned}$$

Similarly $d(v_2v_3) = \mu(v_1v_2)$. Hence $0 < d_1,\ d_2 \leq 1$.

Conversely suppose $0 < d_1, d_2 \leq 1$. Consider a path $v_1v_2v_3$ on three vertices. Assign $\mu(v_1v_2) = d_1$, $\mu(v_2v_3) = d_2$ and any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then (d_1, d_2) is the degree sequence of the fuzzy graph $G : (\sigma, \mu)$ on the path P_3. □

Theorem 3.2. *A decreasing sequence (d_1, d_2, d_3) is the edge degree sequence of a fuzzy graph on a path P_4 on four vertices if and only if either $0 < d_2 = d_3 \leq 1$ and $d_1 \leq 2$ or $0 \leq d_3 < d_1 = d_2 \leq 1$.*

Proof. Suppose (d_1, d_2, d_3) is the edge degree sequence of a fuzzy graph on P_4, say, $v_1v_2v_3v_4$. Then we have $d(v_1v_2) = \mu(v_2v_3) = d(v_3v_4)$. Since $\mu(v_iv_j) \leq 1$ we have $0 < d(v_1v_2), d(v_3v_4) \leq 1$. Also $d(v_2v_3) = \mu(v_1v_2) + \mu(v_3v_4) \leq 1 + 1 = 2$. If $d(v_2v_3) \geq \mu(v_2v_3)$, then we get the desired result by taking $d_1 = d(v_2v_3)$, $d_2 = d(v_1v_2)$, $d_3 = d(v_3v_4)$. If $d(v_2v_3) \leq \mu(v_2v_3)$, then we get the desired result by taking $d_1 = d(v_1v_2)$, $d_2 = d(v_3v_4)$ and $d_3 = d(v_2v_3)$.

Conversely suppose (d_1, d_2, d_3) is a sequence of real numbers with the given hypothesis. If $0 < d_2 = d_3 \leq 1$ and $d_1/2 \leq 1$, let $\mu(v_2v_3) = d_2$ or d_3 and let $\mu(v_1v_2) = \mu(v_3v_4) = d_1/2$. If $0 \leq d_3 < d_1 = d_2 \leq 1$. Let $\mu(v_2v_3) = d_1$ or d_2 and let $\mu(v_1v_2) = \mu(v_3v_4) = d_3/2$. Assign $\sigma(v_i)$ as any value satisfying the condition of a fuzzy graph. Then (d_1, d_2, d_3) is the edge degree sequence of the fuzzy graph (σ, μ) on P_4. □

Let us illustrate the procedure described in the above theorem by the following example.

Example 3.3. *Consider the sequence $S = \{1.6, 0.25, 0.25\}$ which satisfies the hypothesis of the Theorem 3.2 Consider a path P_4 on $v_1v_2v_3v_4$. Assign the value 0.25 as the membership values of the edge v_2v_3. Assign $\mu(v_1v_2) = \mu(v_3v_4) = d_1/2 =$* $1.6/2 = 0.8$. *Also assign any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$ for $i = 1, 2, 3$. Then $G : (\sigma, \mu)$ in Figure 2 is a fuzzy graph on P_4 with S as its degree sequence.*

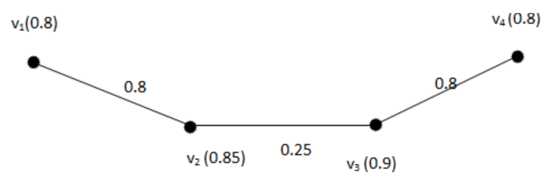

Figure 2:

Theorem 3.4. *A decreasing sequence (d_1, d_2, d_3) is the edge degree sequence of a fuzzy graph on $K_{1,3}$ if and only if $d_j + d_k - 2 \leq d_i < d_j + d_k$ where $i \neq j \neq k$ and $i, j, k \in \{1, 2, 3\}$.*

Proof. Let (d_1, d_2, d_3) be the edge degree sequence of a fuzzy graph on the star $K_{1,3}$, say, v_0, v_1, v_2, v_3 with v_0 as its center. Without loss of generality, assume that $d(v_0v_1) = d_1$; $d(v_0v_2) = d_2$ and $d(v_0v_3) = d_3$. Then we have

$$\mu(v_0v_2) + \mu(v_0v_3) = d_1 \tag{1}$$

$$\mu(v_0v_1) + \mu(v_0v_3) = d_2 \tag{2}$$

$$\mu(v_0v_1) + \mu(v_0v_2) = d_3 \tag{3}$$

(2)+(3)-(1) gives $\mu(v_0v_1) = \frac{d_3 - d_1 + d_2}{2}$

(1)-(2)+(3) gives $\mu(v_0v_2) = \frac{d_1 - d_2 + d_3}{2}$

(1)+(2)-(3) gives $\mu(v_0v_3) = \frac{d_2-d_3+d_1}{2}$

Now $\mu(v_0v_1) > 0 \Rightarrow d_2+d_3-d_1 > 0 \Rightarrow d_2+d_3 > d_1$. Similarly we have $d_2+d_1 > d_3$; $d_1+d_3 > d_2$. Now $\mu(v_0v_1) \leq 1$ gives $d_2+d_3-d_1 \leq 2 \Longrightarrow d_2+d_3-2 \leq d_2$. Similarly $d_2+d_1-2 \leq d_3$and $d_1+d_3-2 \leq d_2$. Hence $d_2+d_3-2 \leq d_1 < d_2+d_3$, $d_3+d_1-2 \leq d_2 < d_3+d_2$ and $d_1+d_2-2 \leq d_3 < d_1+d_2$.

Conversely suppose (d_1, d_2, d_3) is a sequence with the given hypothesis. Consider a fuzzy graph (σ, μ) on $K_{1,3}$ with vertices v_0, v_1, v_2, v_3 in which v_0 is its center. Let $\mu(v_0v_1) = \frac{d_3-d_1+d_2}{2}$, $\mu(v_0v_2) = \frac{d_1-d_2+d_3}{2}$ and $\mu(v_0v_3) = \frac{d_2-d_3+d_1}{2}$. Since $d_j+d_k-2 \leq d_i < d_j+d_k$, we have $0 < \frac{d_j-d_i+d_k}{2} \leq 1$. Therefore $0 < \mu(v_0v_j) \leq 1$ for each $j = 1, 2, 3$. Also

$$\begin{aligned} d(v_0v_1) &= \mu(v_0v_2) + \mu(v_0v_3) \\ &= \frac{d_3+d_1-d_2}{2} + \frac{d_1+d_2-d_3}{2} \\ &= 2(d_1/2) \\ &= d_1 \end{aligned}$$

Similarly $d(v_0v_2) = d_2$ and $d(v_0v_3) = d_3$. Assign any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then (d_1, d_2, d_3) is the edge degree sequence of the fuzzy graph $G : (\sigma, \mu)$ on $K_{1,3}$. □

Example 3.5. *Consider the sequence* $S = (0.5, 0.4, 0.3)$ *which satisfies the hypothesis of the Theorem 3.4 Consider a star* $K_{1,3}$ *on* v_0, v_1, v_2, v_3. *Then* $\mu(v_0v_1) = \frac{d_3-d_1+d_2}{2} = \frac{0.3-0.5+0.4}{2} = 0.1$; $\mu(v_0v_2) = \frac{d_1-d_2+d_3}{2} = \frac{0.5-0.4+0.3}{2} = 0.2$; $\mu(v_0v_3) = \frac{d_2-d_3+d_1}{2} = \frac{0.4-0.3+0.5}{2} = 0.3$. *Also assign any value satisfying the condition of a fuzzy graph as* $\sigma(v_i)$ *for* $i = 1, 2, 3$. *Then* $G : (\sigma, \mu)$ *in Figure 3 is a fuzzy graph on* $K_{1,3}$ *with* S *as its degree sequence.*

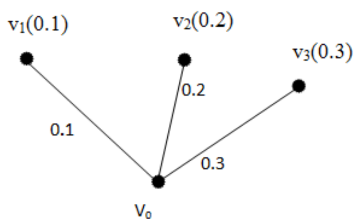

Figure 3:

Theorem 3.6. *A decreasing sequence* (d_1, d_2, d_3) *is the edge degree sequence of a fuzzy graph on a cycle* C_3 *if and only if* $d_j+d_k-2 \leq d_i < d_j+d_k$ *where* $i \neq j \neq k$ *and* $i, j, k \in \{1, 2, 3\}$.

Proof. (d_1, d_2, d_3) is the degree sequence of a fuzzy graph on C_3 say $v_1e_1v_2e_2v_3e_3v_1$. Without loss of generality, assume that $d(e_1) = d_1$; $d(e_2) = d_2$ and $d(e_3) = d_3$. Then we have $d_1 = \mu(v_2v_3)+\mu(v_3v_1)$, $d_2 = \mu(v_3v_1)+\mu(v_1v_2)$, $d_1 = \mu(v_1v_2)+\mu(v_2v_3)$. Proceeding as in Theorem 3.4, we have, $\mu(v_0v_1) = \frac{d_3-d_1+d_2}{2}$, $\mu(v_0v_2) = \frac{d_1-d_2+d_3}{2}$ and $\mu(v_0v_3) = \frac{d_2-d_3+d_1}{2}$. Hence proceeding as Theorem 3.4, $0 < \mu(v_iv_j) \leq 1$, which gives the result.

Conversely suppose (d_1, d_2, d_3) is a sequence with the given hypothesis. Consider the cycle $C_3 : v_1v_2v_3v_1$ and assign $\mu(v_1v_2) = \frac{d_3-d_1+d_2}{2}$, $\mu(v_2v_3) = \frac{d_1-d_2+d_3}{2}$ and $\mu(v_3v_1) = \frac{d_2-d_3+d_1}{2}$. Then proceeding as in Theorem 3.4, we have $d(v_1v_2) = d_1$; $d(v_2v_3) = d_2$ and $d(v_3v_1) = d_3$. Assign any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then $G : (\sigma, \mu)$ is a fuzzy graph on C_3. □

Let us illustrate the procedure described in the above theorem by the following example

Example 3.7. *Consider the sequence* $S = \{0.5, 0.4, 0.3\}$ *which satisfies the hypothesis of the Theorem 3.6. Consider a path* C_3 *on* $v_1v_2v_3$*. Then* $\mu(v_1v_2) = \frac{d_3-d_1+d_2}{2} = \frac{0.3-0.5+0.4}{2} = 0.1$*;* $\mu(v_2v_3) = \frac{d_1-d_2+d_3}{2} = \frac{0.5-\ 0.4+0.3}{2} = 0.2$*;* $\mu(v_1v_3) = \frac{d_2-d_3+\ d_1}{2} = \frac{0.4-0.3+0.5}{2} = 0.3$*. Also assign any value as* $\sigma(v_i)$ *for* $i = 1, 2, 3$ *satisfying the condition of a fuzzy graph. Then* $G : (\sigma, \mu)$ *in Figure 4 is a fuzzy graph on* C_3 *with S as its degree sequence*

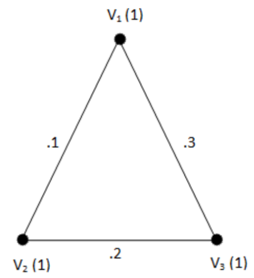

Figure 4:

From the above results we obtain the following theorem, which gives a characterization for a sequence of real numbers with three terms to be an edge degree sequence of a connected fuzzy graph.

Theorem 3.8. *A decreasing sequence* (d_1, d_2, d_3) *is the edge degree sequence of a connected fuzzy graph if and only if* $0 < d_2 = d_3 \leq 1$*;* $d_1 \leq 2$ *or* $0 \leq d_3 < d_1 = d_2 \leq 1$ *or* $d_j + d_k - 2 \leq d_i \leq d_j + d_k$ *where* $i \neq j \neq k$ *and* $i, j, k \in \{1, 2, 3\}$*.*

Proof. Let (d_1, d_2, d_3) be the edge degree sequence of a connected fuzzy graph. Then the possible connected graphs on 3 vertices are P_4, C_3 and $K_{1,3}$. Therefore the result follows from Theorem 3.2, Theorem 3.4 and Theorem 3.6. □

Remark 3.9. *A characterization for the sequence* (d_1, d_2, d_3) *to be the edge degree sequence of various connected fuzzy graphs can also be obtained in the following forms.*

Theorem 3.10. *A decreasing sequence* (d_1, d_2, d_3) *is the edge degree sequence of a fuzzy graph on a path* P_4 *on four vertices if and only if there exists three real numbers* m_1, m_2 *and* m_3 *in* $(0, 1]$ *such that* $d_2 = d_3 = m_i$ *and* $d_1 = m_j + m_k$ *or* $d_1 = d_2 = m_i$ *and* $d_3 = m_j + m_k$*,* $i{\neq}j{\neq}k$*.*

Proof. Let (d_1, d_2, d_3) be the edge degree sequence of a fuzzy graph on P_4, say, $v_1v_2v_3v_4$. Then we have $d(v_1v_2) = \mu(v_2v_3) = d(v_3v_4)$. $d(v_2v_3) = \mu(v_1v_2) + \mu(v_3v_4)$. If m_1, m_2, m_3 are the membership values of the edges of the fuzzy graph G, then they satisfy the required conditions.

Conversely suppose that there exists real numbers, m_1, m_2, m_3 satisfying the given hypothesis. Consider a path P_4: $v_1e_1v_2e_2v_3e_3v_1$ on 4 vertices. Let d_{i_1}, d_{i_2}, d_{i_3} be the rearrangement of the degree sequence d_1, d_2, d_3 so that $m_2 = d_{i_1}$; $m_1 + m_3 = d_{i_2}$; $m_2 = d_{i_3}$. Assign $\mu(v_iv_{i+1}) = m_i$ for every $i = 1, 2, 3$ and any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then (σ, μ) is a fuzzy graph on a cycle P_4 with the edge degree sequence (d_1, d_2, d_3). □

Example 3.11. *Consider the sequence* $S = (1.2, 0.2, 0.2)$ *of real numbers, which satisfies the hypothesis of the Theorem 3.10. There exists numbers* $m_1 = 0.7$*,* $m_2 = 0.5$ *and* $m_3 = 0.2$ *such that* $d_1 = 1.2 = 0.7 + 0.5$ *and* $d_2 = d_3 = m_3 = 0.2$*. Assign* $\mu(v_1v_2) = m_1 = 0.7$*,* $\mu(v_2v_3) = m_2 = 0.2$ *and* $\mu(v_3v_4) = m_3 = 0.5$*. Also assign any value satisfying the condition of a fuzzy graph as* $\sigma(v_i)$ *for* $i = 1, 2, 3$*. Then* (σ, μ) *in Figure 5 is a fuzzy graph on* P_4 *with S as its degree sequence.*

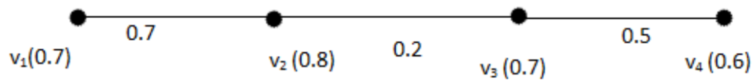

Figure 5:

Theorem 3.12. *A decreasing sequence (d_1, d_2, d_3) is the edge degree sequence of a fuzzy graph on $K_{1,3}$ if and only if there exists three positive real numbers m_1, m_2, m_3 such that each d_i is the sum of two m_j's and each m_j appears in the partition of exactly two d_i's and no pair of m_j's appear in the partition of same pair of d_i.*

Proof. (d_1, d_2, d_3) is the edge degree sequence of a fuzzy graph on $K_{1,3}$ say $v_0v_1v_2v_3$ with v_0 as its center. Then each d_i is the sum of the membership values of the two edges incident on the edge e_i with degree d_i. If m_1, m_2, m_3 are the membership values of the edges of $K_{1,3}$, then they satisfy the required condition.
Conversely suppose that there exists real numbers, m_1, m_2, m_3 satisfying the given hypothesis. Consider a fuzzy graph on $K_{1,3}$ with vertices $v_0v_1v_2v_3$ in which v_0 is its center. Let $d_{i_1}, d_{i_2}, d_{i_3}$ be the rearrangement of the degree sequence d_1, d_2, d_3 so that $m_1 + m_2 = d_{i_1}$; $m_2 + m_3 = d_{i_2}$; $m_3 + m_1 = d_{i_3}$. Assign $mu(v_0v_i) = m_i$ for every $i = 1, 2, 3$ any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then (σ, μ) is a fuzzy graph on $K_{1,3}$ with the edge degree sequence (d_1, d_2, d_3). □

Theorem 3.13. *A decreasing sequence (d_1, d_2, d_3) is the edge degree sequence of a fuzzy graph on a cycle if and only if there exists three positive real numbers m_1, m_2, m_3 such that each d_i is the sum of the two m_i's and each m_j appears in the partition of exactly two d_j's and no pair of m_j's appear in the partition of same pair of d_i.*

Proof. (d_1, d_2, d_3) is the edge degree sequence of a fuzzy graph on a cycle say $v_1v_2v_3v_1$. Then each d_i is the sum of the membership values of the two edges incident on the edge e_i with degree d_i. If m_1, m_2, m_3 are the membership values of the edges of the cycle, then they satisfy the required condition.
Conversely suppose that there exists real numbers, m_1, m_2, m_3 satisfying the given hypothesis. Consider a fuzzy graph on a cycle with vertices $v_1v_2v_3$. Let $d_{i_1}, d_{i_2}, d_{i_3}$ be the rearrangement of the degree sequence d_1, d_2, d_3 so that $m_1 + m_2 = d_{i_1}$; $m_2 + m_3 = d_{i_2}$; $m_3 + m_1 = d_{i_3}$. Assign $\mu(v_0v_i) = m_i$ for every $i = 1, 2, 3$ any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then (σ, μ) is a fuzzy graph on $K_{1,3}$ with the edge degree sequence (d_1, d_2, d_3). □

Let us illustrate the procedure described in the above Theorems 3.12, 3.13 by the following example.

Example 3.14. *Consider the sequence $S = \{0.5, 0.4, 0.3\}$ of real numbers. Partition it as*

$$0.5 = 0.3 + 0.2 \quad \text{(I)}$$

$$0.4 = 0.3 + 0.1 \quad \text{(II)}$$

$$0.3 = 0.2 + 0.1 \quad \text{(III)}$$

which satisfies the hypothesis of the Theorem 3.12 and Theorem 3.13.
Case i: *Consider a triangle C_3 on $v_1v_2v_3$. Assign the values 0.3, 0.2 and 0.1 in the partitions as the member ship values of the edges v_1v_2, v_2v_3, v_1v_3 in any order (Figure 6). Also assign any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$ for $i = 1, 2, 3$. Then $G : (\sigma, \mu)$ in Figure 7 is a fuzzy graph on C_3 with S as its degree sequence.*
Case ii: *Consider a fuzzy graph on $K_{1,3}$ with vertices v_0, v_1, v_2, v_3 in which v_0 is its center. Assign the values 0.3, 0.2 and 0.1 in the partitions as the member ship values of the edges v_0v_1, v_0v_2, v_0v_3 in any order (Figure 8). Also assign any value*

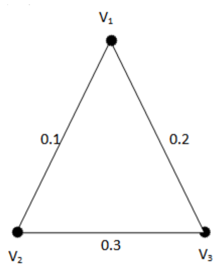

Figure 6:

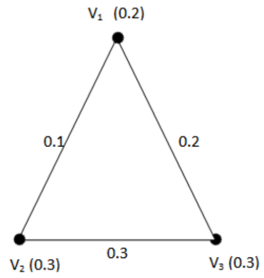

Figure 7:

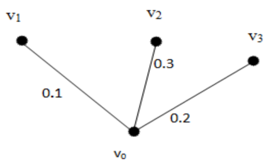

Figure 8:

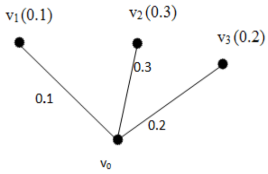

Figure 9:

satisfying the condition of a fuzzy graph as $\sigma(v_i)$ for $i = 1, 2, 3$. Then $G : (\sigma, \mu)$ in Figure 9 is a fuzzy graph on $K_{1,3}$ with S as its degree sequence.

Theorem 3.15. *A decreasing sequence (d_1,d_2,d_3) is the edge degree sequence of a connected fuzzy graph if and only if there exists three positive real numbers m_1, m_2, m_3 such that either two d_i's are equal and has the value m_j for exactly one j and the remaining d_i is equal to the sum of the other two m_j's or each d_i is the sum of the two m_i's such that each m_j appears in the partition of exactly two d_i's and no pair of m_j's appear in the partition of same pair of d_i.*

Proof. Let (d_1, d_2, d_3) be the edge degree sequence of a connected acyclic fuzzy graph. Then the possible connected graphs on 3 vertices are P_4, C_3 and $K_{1,3}$. Therefore the result follows from Theorem 3.10, Theorem 3.12 and Theorem 3.13. ☐

Theorem 3.16. *A decreasing sequence $(d_1, d_2, d_3, \ldots, d_n)$ is the edge degree sequence of a fuzzy graph on a cycle if and only if there exists real numbers $m_1, m_2, m_3, \ldots, m_n$ in (0,1] not necessarily distinct such that for each $i = 1, 2, 3, \ldots, n$, $0 < m_i \leq 1$ and $d_i = m_r + m_s$ for $r \neq s$ in which each m_r appears in the partition of exactly two d_i's.*

Proof. Let $(d_1, d_2, d_3, \ldots, d_n)$ be the degree sequence of a fuzzy graph G on a cycle. Then each d_i is the sum of the membership values of the two edges incident on the edge with edge degree d_i. If $m_1, m_2, m_3, \ldots, m_n$ are the membership values of the n edges of the cycle G, then they satisfy the required conditions.

Conversely, suppose that there exists real numbers, $m_1, m_2, m_3, \ldots, m_n$ satisfying the given hypothesis. Consider a cycle $C : v_1 v_2 v_3 v_4 \ldots v_n v_1$ on n vertices. Let $d_{i_1}, d_{i_2}, d_{i_3}, \ldots, d_{i_n}$ be the rearrangement of the degree sequence $d_1, d_2, d_3, \ldots, d_n$ so that

$$m_n + m_2 = d_{i_1}$$

$$m_1 + m_3 = d_{i_2}$$

$$\vdots$$

$$m_{n-1} + m_1 = d_{i_n}$$

Assign $\mu(v_iv_{i+1}) = m_i$ for every $i = 1, 2, 3, \ldots, n$ where $v_{n+1} = v_1$ and any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then $G : (\sigma, \mu)$ is a fuzzy graph on a cycle C with the edge degree sequence $(d_1, d_2, d_3, \ldots, d_n)$. □

Theorem 3.17. *A decreasing sequence $(d_1, d_2, d_3, \ldots, d_n)$ is the edge degree sequence of a fuzzy graph on a path if and only if there exists real numbers $m_1, m_2, m_3, \ldots, m_n$ in (0, 1] not necessarily distinct, which includes exactly two d_i' s, say d_j and d_k, such that $0 < d_j$, $d_k \leq 1$ and for all the remaining $n-2$ edge degrees, $d_i = m_r + m_s$; $r \neq s$ such that each m_j appears in the partition of exactly two d_i' s including d_j and d_k.*

Proof. Let $(d_1, d_2, d_3, \ldots, d_n)$ be the edge degree sequence of a fuzzy graph G on a path $v_1e_1v_2e_2v_3e_3\ldots e_nv_ne_{n+1}v_{n+1}$ on $n+1$ vertices. Let $m_i = \mu(v_iv_{i+1})$, $i = 1, 2, 3, \ldots, n$. Then $0 < m_i \leq 1$ for all $i = 1, 2, 3, \ldots, n$. Also $d(e_1) = \mu(v_2v_3) = m_2$ and $d_n = d(e_n) = \mu(v_{n-1}v_n) = m_n$. $d(e_2) = \mu(v_1v_2) + \mu(v_3v_4) = m_1 + m_3$. Similarly $d(e_3) = m_2 + m_4, \ldots, d(e_{n-1}) = m_{n-2} + m_n$. Hence $m_1, m_2, m_3, \ldots, m_n$ satisfy the required conditions.
Conversely, suppose that there exists real numbers, $m_1, m_2, m_3, \ldots, m_n$ satisfying the given hypothesis. Consider a path $P : v_1v_2\ldots v_n$. on n vertices. Let $d_{i_1}, d_{i_2}, d_{i_3}, \ldots, d_{i_n}$ be the rearrangement of the degree sequence $d_1, d_2, d_3, \ldots, d_n$ so that

$$\begin{aligned} m_2 &= d_{i_1} \\ m_1 + m_3 &= d_{i_2} \\ &\vdots \\ m_{n-2} + m_n &= d_{i_{n-1}} \\ m_{n-1} &= d_{i_n} \end{aligned}$$

Assign $\mu(v_iv_{i+1}) = m_i$ for every $i = 1, 2, 3, \ldots, n$ and any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then $G : (\sigma, \mu)$ is a fuzzy graph on the path P with the degree sequence $(d_1, d_2, d_3, \ldots, d_n)$. □

4. Conclusion

In this paper, we have discussed some properties of edge degree sequence of a fuzzy graph. We have obtained necessary and sufficient conditions for a sequence of two or three real numbers to be an edge degree sequence of a fuzzy graph. Also we have obtained necessary and sufficient conditions for a finite sequence of real numbers to be an edge degree sequence of a fuzzy graph on a path and cycle. They will be helpful in the study of fuzzy graph and its applications.

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