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Laminar Boundary Layer Flow of Non-Newtonian Power Law Fluid Near Stagnation Point with Suction/Injection

Research Article

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Abstract: Laminar boundary layer flow of Non-Newtonian Power law fluid near stagnation point with Suction/Injection effects has been considered. The governing equations of continuity and momentum are transformed into ordinary differential equations using similarity transformations. The equations are solved by using method of successive approximations starting with zeroth approximation. For $n = 1$ the results tallies with Corresponding results for Newtonian fluids. Boundary layer parameters and Velocity profiles have been drawn for different values of parameter n; Suction/Injection parameter f_w shows the behavior of power law fluids. The Skin friction coefficient and boundary layer parameters obtained are very close to exact results, although the method employed gives approximate results.

Keywords: Boundary layer, power law fluids, successive approximations, stagnation point, Suction/Injection, velocity profiles.

1. Introduction

Boundary layer flows of viscous, incompressible fluid past semi infinite flat plate were studied by Blassius [1], Howarth [2]. Tsou et.al [3] studied flow and heat transfer in the boundary layer flow on a continuously moving flat surface. Two dimensional boundary layer flow of viscous fluid near stagnation point was studied by Hiemenz[4]and then the solution has been improved by Howarth [4], Rott [5]. The study of the stagnation point flows with suction was made by Schlichting and Bassman[6], Mishra and Choudhary [7], Smith [8]. The MHD aspects of stagnation point flow has been considered by Dutta and Nath[9]. In case of non-Newtonian fluid, Srivastav [10] studied the problem for Second order fluid. The boundary layer flows of power law fluid near stagnation point were solved by Maiti [11]. By using the method of Successive approximation, starting with zeroth approximation used by B P Jadhav [12], we have obtained the solution to boundary layer flow of power law fluid near stagnation point.The aim of this paper is to study the effects of the suction/injection on the stagnation point flow of power law fluid.

2. Mathematical Analysis

Consider a steady, two dimensional flow of an electrically conducting non-Newtonian power law fluid past semi infinite plate near stagnation point. The boundary layer equations are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \gamma\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{2}$$

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With boundary conditions

$$u = 0,\ v = v_w \ \text{at} \ y = 0 \ \text{and} \ u = U(x),\ v = 0 \ \text{as} \ y \to \infty. \tag{3}$$

At the forward stagnation point the potential velocity U is given by $U(x) = u_1 x$, where u_1 is constant.

3. Solution of the Problem

To solve the boundary layer equations under the conditions (3), we introduce a Similarity transformations

$$\eta = y\left(\frac{U^{2-n}}{\gamma x}\right)^{\frac{1}{n+1}}, \quad \emptyset\,(\eta) = \left(\gamma x U^{2n-1}\right)^{\frac{1}{n+1}} f(\eta) \tag{4}$$

Such that

$$u = \frac{\partial \emptyset}{\partial x}, \quad v = -\frac{\partial \emptyset}{\partial y} \tag{5}$$

Substituting these values in the equation (2), it reduces to

$$n(f^{''})^{n-1} f^{'''} + \left(\frac{2n}{n+1}\right) f f^{''} + 1 - f'^2 = 0 \tag{6}$$

For $n = 1$, it reduces to

$$f^{'''} + f f^{''} + 1 - f'^2 = 0 \tag{7}$$

This was same as obtained by Hiemenz [4] for the Newtonian fluid. Subject to boundary conditions,

$$f\,(0) = f_w,\ f'\,(0) = 0,\ f'\,(\infty) = 1 \tag{8}$$

Where $f_w = -\frac{(n+1)v_w}{2nU} R_e^{\frac{1}{n+1}}$ is suction/injection parameter, $f_w > 0$ for suction and $f_w < 0$ for injection and $R_e = \frac{U^{2-n}x^n}{\gamma}$ is modified Reynolds number. We shall solve the equation (6) under the boundary conditions (7) by using the method of successive approximations starting with zeroth approximation. Taking the zeroth approximation as

$$f(\eta) = f_w + \eta + \frac{1}{\beta} e^{-\beta\eta} - \frac{1}{\beta} \tag{9}$$

Where β is arbitrary constant to be determined such that for the first approximation $f_1'\,(\infty) = 1$, i. e. β is real root of the equation

$$\beta^{n+1} + \frac{4}{(n+1)\,(n-2)^3} - \frac{2(1 + n f_w \beta)}{n\,(n+1)\,(n-2)^2} - \frac{n-1}{n\,(n+1)\,(n-3)^2} = 0 \tag{10}$$

The different successive approximations can be obtained from following relation

$$n f_i^{'''} = -(f_{i-1}^{''})^{1-n} \left[\frac{2n}{n+1} f(f_{i-1}^{''}) + 1 - f_{i-1}'^2\right], \quad i = 1, 2, 3, \ldots \tag{11}$$

For the first approximation, we have,

$$n f_1^{'''} = -(f_0^{''})^{1-n} \left[\frac{2n}{n+1} f_0(f_0^{''}) + 1 - f'^2_0\right] \tag{12}$$

Integrating equation (12) together with boundary conditions (8) and equation (9) gives

$$f_1'(\eta) = (A_1 - A_2\eta - A_3)e^{(n-2)\beta\eta} - (A_4)e^{(n-3)\beta\eta} + 1 \tag{13}$$

$$f_1(\eta) = \left[\frac{(A_1 - A_2\eta - A_3)\,(n-2)\,\beta + A_2}{(n-2)^2\beta^2}\right] e^{(n-2)\beta\eta} - \left(\frac{A_4}{(n-3)\beta}\right) e^{(n-3)\beta\eta} + \eta + C_1 \tag{14}$$

$$A_1 = \frac{4}{(n+1)\,(n-2)^3\beta^{n+1}},\ A_2 = \frac{2}{(n+1)\,(n-2)^2\beta^n},\ A_3 = \frac{2(1+n\,f_w\,\beta)}{n\,(n+1)\,(n-2)^2\beta^{n+1}},$$

$$A_4 = \frac{n-1}{n\,(n+1)\,(n-3)^2\beta^{n+1}},\ \ C_1 = f_w - \frac{(A_1 - A_2)}{(n-2)\beta} - \frac{A_2}{(n-2)^2\beta^2} + \frac{(A_4)}{(n-3)\beta} \tag{15}$$

4. Discussions

For different values of n and suction/injection parameter f_w, the values of β are obtained from the equation (10) using Matlab 2012. For different values of n and f_w skin friction coefficient $c_f{}^* = [f_1''(0)]^n$ is obtained. Where,

$$f_1''(0) = (A1 - A3)\,(n-2)\beta - A2 - A4(n-3)\beta \tag{16}$$

The displacement thickness δ_1 and momentum thickness δ_2 are given by

$$\delta_1 = \int_0^\infty (1 - f_1)dy, \qquad \delta_2 = \int_0^\infty f_1'(1 - f_1)dy$$

$$\delta_1{}^* = \frac{\delta_1}{x} R_e{}^{\frac{1}{n+1}}, \qquad \delta_2{}^* = \frac{\delta_2}{x} R_e{}^{\frac{1}{n+1}}$$

Hence,

$$\delta_1^* = \frac{(A_1 - A_3)}{(n-2)\beta} + \frac{A_2}{(n-2)^2\beta^2} - \frac{(A_4)}{(n-3)\beta}, \tag{17}$$

$$\delta_2{}^* = \frac{(A_1 - A_3)^2}{2(n-2)\beta} + \frac{A_2(A_1 - A_3)}{2(n-2)^2\beta^2} + \frac{A_2{}^2}{4(n-2)^3\beta^3} - \frac{2(A_4)(A_1 - A_3)}{(2n-5)\beta} - \frac{2(A_2)(A_4)}{(2n-5)^2\beta^2} + \frac{A_4{}^2}{2(n-3)\beta} + \frac{(A_1 - A_3)}{(n-2)\beta} + \frac{A_2}{(n-2)^2\beta^2} - \frac{(A_4)}{(n-3)\beta} \tag{18}$$

For different values of n and suction/injection parameter f_w, the boundary layer parameters are calculated which have been tabulated in Table 1

It has been observed that for fixed value of n, the increase in suction leads to decrease in thicknesses δ_1^* and δ_2^* and increase in the skin friction c_f^*, while just opposite nature occurs in case of injection. The effect of suction is to decrease the ratio of thicknesses and injection leads to increase in the ratio of thicknesses. It may be noted that for fixed value of f_w, increase in flow index n increases the thicknesses and their ratio. It is interesting to know that as n increases, the skin friction decreases for the flow without suction/injection .For the flow with suction increases n results in increase in skin friction for the range $0.5 \le n < 1$, but for $n \ge 1$, the skin friction is decreasing. In case of flow with injection, as n increases the skin friction decreases. The velocity profiles drawn for different values of suction/injection parameter f_w and flow index n are shown in figs 1-5. For fixed n; increase in suction leads to increase in velocity and increase in injection leads to decrease in velocity. For fixed f_w, increase in n decreases the velocity except at $n = 1$. The velocity of Newtonian fluid is always higher than the non-Newtonian fluid for all values of suction/injection parameter considered. The comparison of velocity profiles for different values of n and f_w has been shown in fig.6.

n	f_w	$\delta_1{}^*$	$\delta_2{}^*$	δ_1/δ_2	$c_f{}^*$
	0.0	**0.5424**	**0.2638**	**2.056**	**1.3069**
	0.5	**0.4398**	**0.2159**	**2.037**	**1.4718**
0.5	**1.0**	**0.3543**	**0.1751**	**2.024**	**1.6548**
	-0.5	**0.6639**	**0.3187**	**2.083**	**1.1566**
	-1.0	**0.8059**	**0.3803**	**2.119**	**1.0175**
	0.0	**0.6927**	**0.3342**	**2.073**	**1.2400**
	0.5	**0.9625**	**0.2751**	**2.044**	**1.5145**
0.8	**1.0**	**0.4608**	**0.2273**	**2.027**	**1.8104**
	-0.5	**0.8587**	**0.4656**	**2.117**	**0.9851**
	-1.0	**1.0684**	**0.4890**	**2.185**	**0.7463**
	0.0	**0.7698**	**0.3689**	**2.087**	**1.1547**
	0.5	**0.6250**	**0.3047**	**2.051**	**1.5000**
1.0	**1.0**	**0.5161**	**0.2542**	**2.030**	**1.8685**
	-0.5	**0.9630**	**0.4486**	**2.147**	**0.8333**
	-1.0	**1.2198**	**0.5433**	**2.245**	**0.5351**
	0.0	**0.9644**	**0.4522**	**2.132**	**0.7948**
	0.5	**0.7870**	**03791**	**2.076**	**1.2376**
1.5	**1.0**	**0.6621**	**0.3238**	**2.045**	**1.7130**
	-0.5	**1.2231**	**0.5460**	**2.240**	**0.4030**
	-1.0	**1.6074**	**0.6560**	**2.450**	**0.0986**
	0.0	**1.1336**	**0.5239**	**2.164**	**0.5101**
	0.5	**0.9372**	**0.4472**	**2.096**	**0.8895**
1.8	**1.0**	**0.8001**	**0.3886**	**2.059**	**1.3080**
	-0.5	**1.4254**	**0.6213**	**2.294**	**0.1987**
	-1.0	**1.8719**	**0.7305**	**2.563**	**0.0126**

Table 1:

5. Conclusions

The method employed gives good agreement with the results obtained by Himenz for Newtonian fluid. For n=1, the boundary layer parameters obtained are very close to exact results. The behavior of the non-Newtonian power law fluid can be well judged by this method.

1. The effect of suction is on stagnation point flow is to decrease the thickness and increase the skin friction and velocity.
2. Increase in injection leads to increase the thickness and decrease the skin friction and velocity.
3. For higher suction values, the skin friction for the Newtonian fluid is greater than the non-Newtonian fluids.
4. For zero suction/injection, increase in the flow index yields increase in the thicknesses and decrease in the skin friction and velocity.
5. For the flow with injection, increase in the flow index n leads to increase in thicknesses and decrease in the skin friction and velocity.

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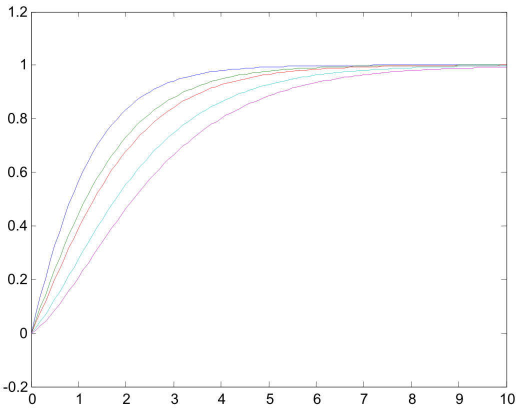

Figure 1: Velocity Profiles for $f_w = 0.0$, $n = 0.5, 0.8, 1.0, 1.5, 1.8$

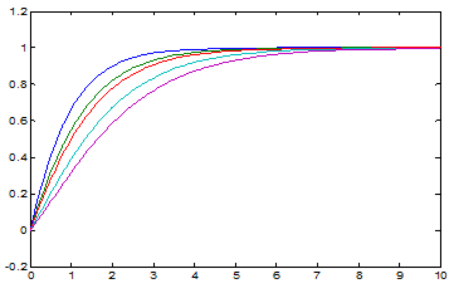

Figure 2: Velocity Profiles for $f_w = 0.5$, n=0.5,0.8,1.0,1.5,1.8

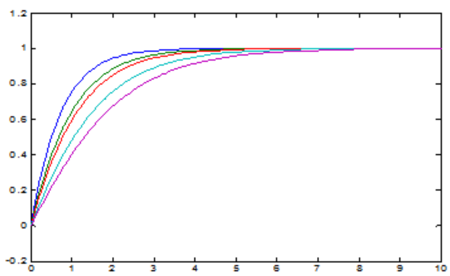

Figure 3: Velocity Profiles for $f_w = 1.0$, n=0.5,0.8,1.0,1.5,1.8

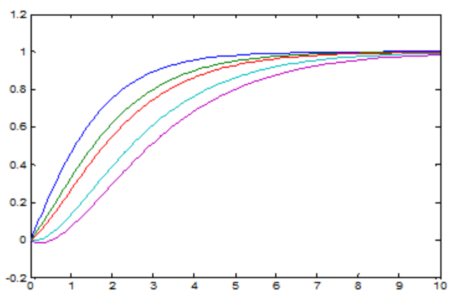

Figure 4: Velocity Profiles for $f_w = -0.5$, n=0.5,0.8,1.0,1.5,1.8

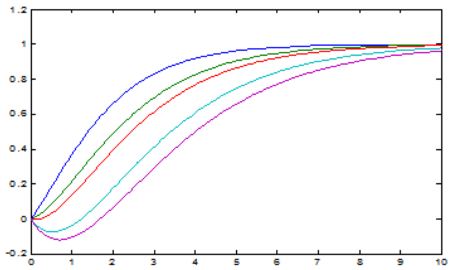

Figure 5: Velocity Profiles for $f_w = -1.0$, n=0.5,0.8,1.0,1.5,1.8

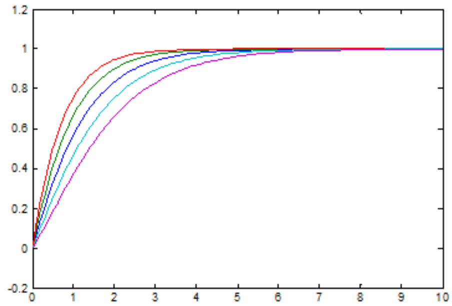

Figure 6: Velocity Profiles for $n = 0.5$, f_w=0.0,0.5,1.0,-0.5,-1.0

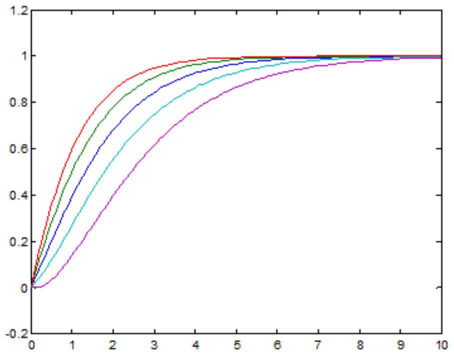

Figure 7: Velocity Profiles for $n = 1.0$, f_w=0.0,0.5,1.0,-0.5,-1.0

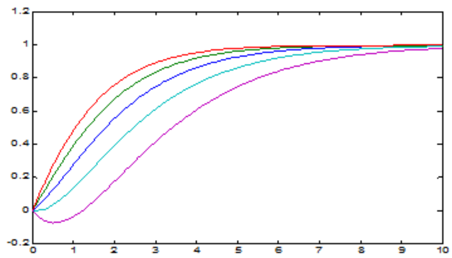

Figure 8: Velocity Profiles for $n = 1.5$, f_w=0.0,0.5,1.0,-0.5,-1.0